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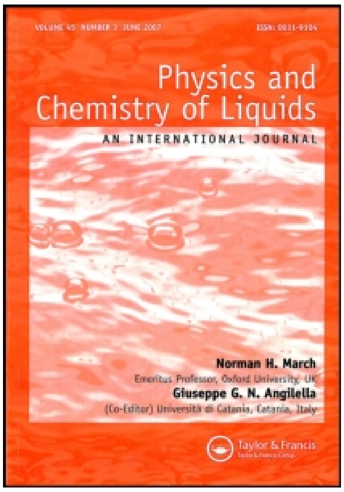

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Letter

Roton excitations in Bose–Einstein condensates and a fluid–solid transition

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Several authors have recently discussed the existence of roton excitations in Bose–Einstein condensates (BECs) and considered how a roton dip in the dispersion curve of elementary excitations may be related to the formation of a spatially modulated ground state. Here attention is drawn to a theoretical study of Minguzzi *et al.* on interatomic correlations in a BEC from dipole–dipole interactions induced by laser light of increasing intensity. Attractive interactions in superfluid ^{4}He and repulsive interactions in 'untuned' BECs are then compared and contrasted, the experiments of Woods and Cowley and of Greiner *et al.* providing the focus respectively. It is stressed that, contrary to a very recent assertion by Nazario and Santiago, ^{4}He is crucially different from BECs at the lowest temperatures.

Keywords: Bose–Einstein condensates; Dipole–dipole interactions; Elementary excitations

It has been shown in the recent literature that the emergence of a roton dip in the dispersion relation of elementary excitations in a gaseous Bose–Einstein condensate (BEC) can be driven by interatomic correlations due to long-range interactions between dipoles, either permanent [1] or induced by irradiation with a far-off-resonance laser [2]. This feature in the excitation spectrum of a BEC may in turn lead to a quantum phase transition into a spatially modulated ground state [3,4]. Nazario and Santiago [5] have more recently proposed that roton excitations in a BEC may be signatures of proximity to a Mott insulating phase. Here their study is first related to previous theoretical conclusions drawn by Minguzzi *et al.* [3]. These authors drew on the work of O'Dell *et al.* [2] and interpreted the BEC structure factor $S(k, I)$ driven by laser intensity I as heralding a fluid–solid transition at a critical height of its main peak.

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We deal then not only with common ground between BECs and the strongly coupled superfluid ^{4}He, which is already stressed by Nazario and Santiago [5], but supplement their discussion by drawing attention to essential differences between the two systems, neutron scattering experiments of Woods and Cowley [6] on the dynamical structure factor $S(k, \omega)$ of ^{4}He under pressure being cited.

As a little further background, in [3] the structure factor $S(k, I)$ in the study of O'Dell *et al.* [2] was divided into the sum of two parts: a background contribution, $S_0(k)$ say, plus a term driven by the laser-induced dipole–dipole interaction, which influences only a restricted range of wave number k in the structure factor. Thus, in [3], the explicit model adopted was

$$S(k, I) = S_0(k) + S_I(k). \tag{1}$$

In figure 1, a plot of the main-peak height of $S(k, I)$ is redrawn from [3] and has a behavior leading to an infinite value reflecting an instability of the fluid state at $I = 0.654\,\mathrm{W\,cm^{-2}}$. Minguzzi *et al.* [3] pointed out, by appealing to the so-called density wave theory of freezing, that such an instability will be anticipated by a first-order transition to a spatially modulated ground state, the critical height of the main peak of the structure factor being about 2.8 for freezing of a classical liquid [7]. In a quantal fluid, it was proposed in [3], based at least partly on the neutron scattering data of Woods and Cowley [6] on ^{4}He under pressure, that the peak height at which the fluid–solid transition would occur should be about a half of the classical liquid value, say 1.4. Figure 2 shows again, following [3], the way the collective mode frequency $\omega(k, I) \propto k^2/S(k, I)$ develops a roton-like minimum as I is increased toward the proposed 'freezing' transition.

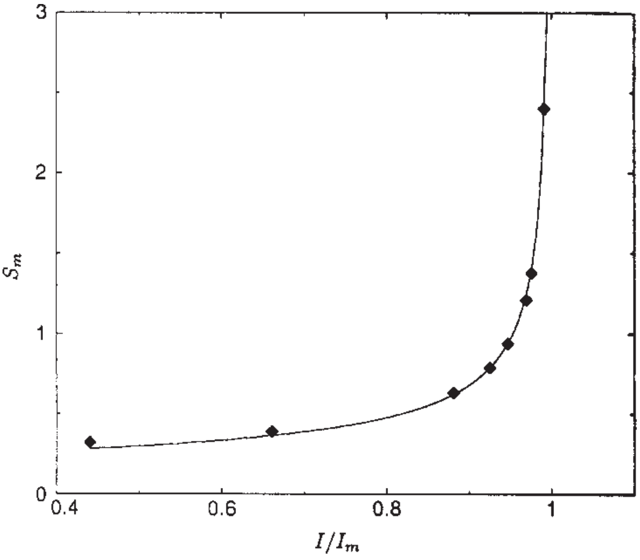

Figure 1. Peak height S_m of the structure factor $S(k)$ in equation (1) (in arbitrary units) plotted as a function of laser intensity I (in units of the intensità I_m for which the roton gap vanishes). Diamonds denote values using the model and system parameters of O'Dell *et al.* [2]. Solid line represents the function $S_m(I) = S_0/(1 - I/I_m)^{\alpha}$ with $\alpha \sim 1/2$. (Redrawn from Minguzzi *et al.* [3].)

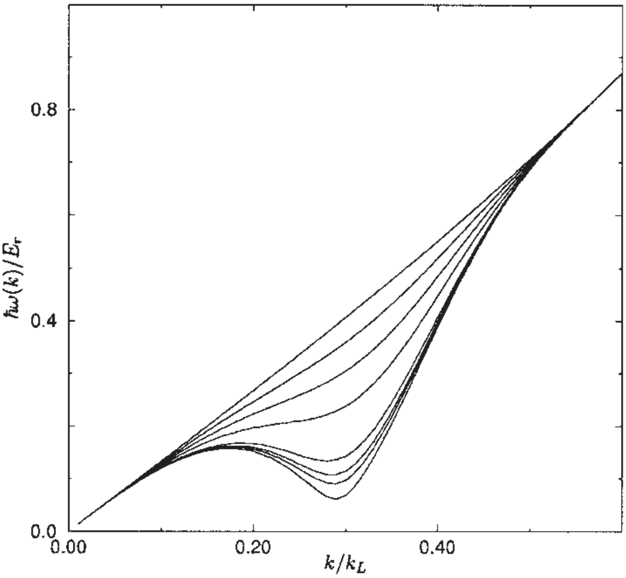

Figure 2. Shows dispersion relation $\omega(k)$ of elementary excitations, with attention focusing on roton features as intensity I is increased. $\hbar\omega(k)$ is in units of recoil energy E_r, while wave number k is in units of laser wave number k_L. As in figure 1, this is for a dipolar gas with model and parameters of [2]. From top to bottom, different curves correspond to values of laser intensity ratio I/I_m given by $I/I_m = 0$, 0.22, 0.44, 0.66, 0.88, 0.92, 0.947, and 0.975. (Redrawn from Minguzzi *et al.* [3].)

Let us turn, more specifically, to the identification by Nazario and Santiago [5] of roton excitations as heralds of a Mott insulator. Then it is highly relevant to draw attention to the landmark experiment of Greiner *et al.* [8]. These authors employed a web of laser beams to create an optical lattice taking the form of an energy landscape of mountains and valleys. They were then able to reversibly switch a gas of ^{87}Rb atoms from a superfluid to an insulating state by control of the laser intensity. Starting from a superfluid BEC in which the atoms can tunnel between valleys, an increase of the laser intensity enhances the optical-lattice barriers and forces the gas into an insulating state. The interpretation of the experiment based, for example, on figure 1 of Stoof's review article [9] is bound up with the Heisenberg Uncertainty Principle, which prevents simultaneous knowledge of the number of atoms in a particular valley and of the phase of the condensate wave function in that same valley. Thus, when the gaseous assembly is superfluid, long-range phase coherence requires that the number of atoms in each lattice well be subject to considerable fluctuations. This situation is reversed in the insulating state, the number of atoms in each well now being fixed, and therefore the phase must change randomly from one well to another. The existence of the insulating state as driven by increasingly strong short-range repulsions was anticipated in the theoretical work on the so-called Bose–Hubbard model by Fisher *et al.* [10], who termed it a Mott insulator as taken up by Nazario and Santiago [5].

However, these authors [5] then make some surprisingly general assertions which we show below must be subject to important qualifications. In their Abstract, they write "BECs are superfluids just like bosonic helium is and all interacting bosonic fluids are expected to be at low enough temperatures". In [5], "universality" is also claimed between BECs and ^{4}He. Following the review in this Journal by Minguzzi *et al.* [11], and prompted by the above assertions made in [5], some points need to be re-emphasized, namely:

(i) There is incontrovertible evidence that superfluidity and essentially complete Bose–Einstein condensation coexist in ultra-cold gases of bosonic alkali atoms in traps.

(ii) In contrast to (i), superfluidity is dramatically in evidence in ^{4}He but, while the superfluid fraction is essentially 100% below 1 K [12], only 7% of the atoms are in a condensate. This value is arrived at both from neutron scattering experiments [13] and from Diffusion Monte Carlo calculations [14]. In particular, Minguzzi *et al.* [11] comment that it is not easy to see how such a small condensate fraction could be responsible for the dramatic manifestations of superfluidity. In fact, the superfluid fraction and the condensate fraction explore the one-body density matrix in entirely different domains, so that Bose–Einstein condensation and superfluidity are two distinct concepts (though presumably related through deeper topological properties of the many-body wave functions).

(iii) Following the facts recorded in (ii) above, superfluidity without Bose–Einstein condensation is known to occur even at zero temperature in a two-dimensional $\ln(r)$ Bose gas [15,16].

To complete this Letter, we wish first to stress that the difference between repulsive (untuned) interactions in BECs and attractive interactions in superfluid ^{4}He leads to crucial differences in physical properties and next to point out some directions for future progress. As to the first area, let us approach this by a 'thought' experiment bearing on the neutron inelastic scattering measurements of Woods and Cowley [6] on ^{4}He. As we reduce the pressure in such experiments from a suitably high value at $T=0$, solid helium will melt into the superfluid phase, the order parameter becoming the condensate density: from zero in the solid to less than 7% condensate fraction in the liquid at $T=0$. This is to be contrasted with the discussion above, for repulsive interactions, of the BECs in an optical lattice [8–10]. In fact, the many-body wave function is fundamentally different in superfluid ^{4}He and in BECs, this difference being ultimately related to the fact that superfluid ^{4}He is a relatively dense liquid whereas the BECs are dilute gases in which (in the absence of long-range interactions such as due to dipoles) the mean interparticle distance is enormously greater than the range of the interatomic forces [17]. As a consequence, only repulsive s-wave scattering between pairs of atoms plays a role here, whereas attractive (van der Waals) interactions are crucial in the former quantum fluid. The essential role of both two-atom and three-atom correlations in determining the dispersion relation of roton excitations in liquid ^{4}He was already emphasized in the seminal work of Feynman and Cohen [18].

As to future directions, in the context of repulsive interactions in a single-component Bose gas, the very recent study of the dynamical structure factor $S(k,\omega)$ in optical lattices by Roth and Burnett [19] is worthy of considerable development and could shed further light on some issues raised here. The experimental observation of a superfluid-to-insulator transition in one-dimensional lattices [20] again seems fruitful for further studies, both theoretically and experimentally. As to attractive interactions, especially in low-dimensional quantum fluids, there is evident interest in re-opening the early work of Lieb and Liniger [21]. Finally, returning to ^{4}He, it seems clear that the experimental proof, which now exists for stable dimers and trimers in the gaseous phase, makes worthwhile the further study of Efimov states [22–24] at the lowest possible temperatures.

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